Grade 6 Math Circles

Nov 1/2/3, 2022

BCC Prep Solutions

Lesson Solutions

Bear Selection (BCC Grade 5/6 2020)

Story

Ren is allowed to bring one of his four teddy bears to school. Ren brings the bear that has a star on one of its feet, and is wearing a scarf or a bow, but not glasses.

Question

Which bear did Ren bring to school?

(A) 

(B) 

(C) 

(D) 

Solution - Bear Selection (BCC Grade 5/6 2020)

Answer

(D) 

Explanation of Answer

Ren did not choose the bear in Option A or the bear in Option C because those bears have glasses. Ren did not choose the bear in Option B because that bear does not have a star on one of its feet.

Ren chose the bear in Option D. The bear in Option D has a star on one of its feet, is wearing a scarf, and does not have glasses.

Comments

Notice how we solved this question by ruling out the options that weren't possible. In this case, we were left with one option which was the correct answer. In some problems, we can't rule out every wrong answer, but ruling out some can help us focus on the leftover possibilities and, if we need to guess, we'll have a better chance of guessing the correct answer.

Connections to Computer Science

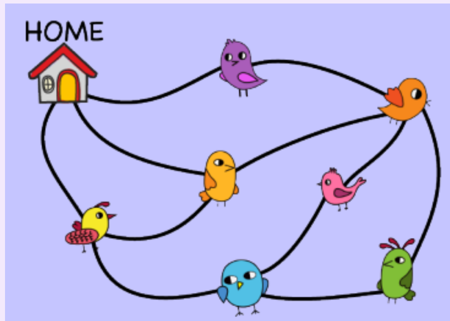
This problem requires the use of *boolean logic*. Each of Ren's requirements is expressed as a *boolean statement*. A boolean statement is either *true* or *false*. The three individual statements are then combined together using the *boolean operators* AND, OR, and NOT. An AND statement is true if all statements it connects are true. An OR statement is true if at least one of the statements it connects is true. A NOT statement is true if the statement it is referring to is false. We can think of Ren's set of requirements as the following *boolean expression* which combines the three statements above:

(star on one of its feet) AND (wearing a scarf OR wearing a bow) AND (NOT has glasses)

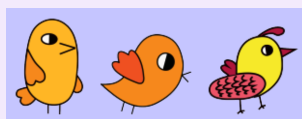
Bird Watching (BCC Grade 5/6 2020)

Story

A family went for a walk. They started from their home and walked along some paths, eventually returning home. They did not walk on any path more than once.

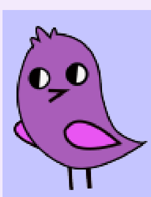
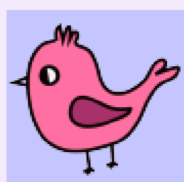


During their walk they saw exactly four birds. Three of the four birds they saw are shown below:



Question

Which other bird must they have seen?

(A)  (B)  (C)  (D) 

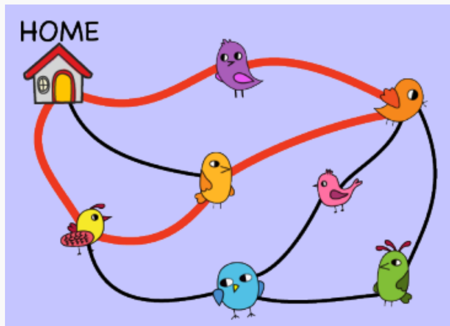
Solution - Bird Watching (BCC Grade 5/6 2020)

Answer

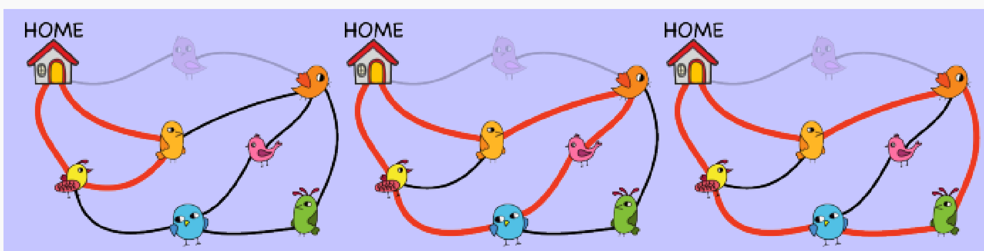
(A) 

Explanation of Answer

The map below shows a route the family could have taken that passes exactly four birds and includes the three birds that they saw. The fourth bird seen along this route is the bird in Option A.



To be completely sure that the family could not have seen any of the other birds, consider what happens if they do not see the bird in Option A. In order to avoid seeing the bird in Option A the family must not take either of the two paths that are adjacent to the bird in Option A. Of the remaining paths, there are three possible routes the family could take.



In the first possible route the family would see exactly two birds. In the second and third possible routes the family would see exactly five birds. There is no route that allows the family to see exactly four birds.

Therefore, the family must have seen the bird in Option A.

Connections to Computer Science

In computer science, this type of diagram is called a *graph*. The birds are the *vertices* and the paths are the *edges* of the graph.

Different computer algorithms can be used to find *cycles* in a graph. A cycle is a sequence of edges that can be followed starting from a vertex and ending at this same vertex. In our task, we wish to find a cycle starting from the family's home. The cycle must include exactly four birds, where three of the birds are specified.

Market Exchange (BCC Grade 5/6 2020)

Story

A beaver goes to a market to trade items. It has one carrot  but needs one fir tree . Each stall of the market allows a different trade as shown:

Stall	Give	Get
P		
Q		
R		
S		
T		
U		
V		
W		

Question

Which of the following sequences of stalls should the beaver visit in order to trade its carrot  for one fir tree ?

(A) P, Q, T (B) W, T, U (C) S, V, U (D) S, R, U

Solution - Market Exchange (BCC Grade 5/6 2020)

Answer

(C) S,V,U

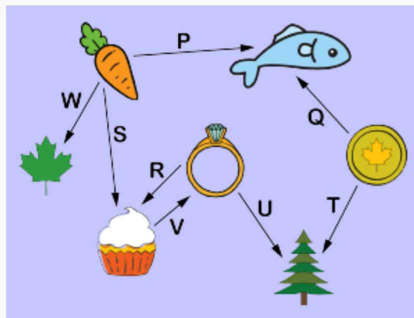
Explanation of Answer

At stall S the beaver can trade the carrot  for a cupcake .

At stall V the beaver can trade the cupcake  for a ring .

At stall U the beaver can trade the ring  for a fir tree .

To consider the other options, the following diagram can help us understand the possible trades at various stalls. An arrow from item X to item Y, labelled Z, means that at stall Z the item X can be traded for item Y.



Notice how you can now trace the connection from a carrot to a fir tree. Also, notice that when starting at the carrot, the only way to reach the fir tree is by following the arrows labelled S, V, and U, in that order.

Comments

Using a diagram like the one above can help with understanding the solution. In this case, we can easily see that there is only one path from a carrot to a fir tree.

Connections to Computer Science

The ways in which items are traded at this market can be modelled by a *directed graph*. We can represent the items as the *vertices* of the directed graph and a possible trade of one item for another as a *(directed) edge*. In the diagram, the edges are the arrows labelled with the particular stall that can trade one item for another.

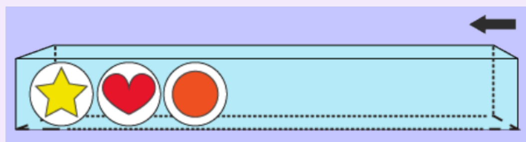
The reason we say "directed" is because the trade is only in one direction. For example, if item A can be traded for item B at stall X, then we have a directed edge from item A to item B labelled with X. In this case, we call B a *neighbour* of A.

In this problem, we want to find a *directed path* from the carrot to the fir tree. One way to do this is to perform a *breadth-first search*, where we first determine all the neighbours of the carrot, and then repeatedly determine the neighbours of each of those neighbours, and so on.

Box of Balls (BCC Grade 5/6 2019)

Story

A beaver has a box with an opening on the right-hand side.



At any time, the beaver can take out the rightmost ball from the box, or put in a new ball from the right. For example, if the beaver wants  in between  and , it needs to take out , put in , and then put in .

Now suppose the beaver has five balls in the box as shown, and two balls ( and ) out of the box.

The beaver wants the balls in the box to be in the order: , , , , , , .

Question

What should the beaver do?

(A) Take out , take out , put in , put in , put in , and then put in .

(B) Take out , take out , put in , put in , put in , and then put in .

(C) Take out , put in , put in , and then put in .

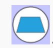
(D) Take out , take out , put in , put in , put in , and then put in .

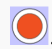
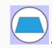
Solution - Box of Balls (BCC Grade 5/6 2019)

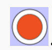
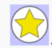
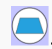
Answer

(B) Take out , take out , put in , put in , put in , and then put in .

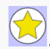
Explanation of Answer

Option A is incorrect. These actions will lead to the result of , , , , , , .

Option C is incorrect. These actions will lead to the result of , , , , , , .

Option D is incorrect. These actions will lead to the result of , , , , , , .

Option B is correct. After the series of actions the balls will be in the beaver's desired order.

In the box, the first three balls on the left are , , and . These satisfy the order given in the question, so there is no need to move these balls. The next ball should be , but it is outside of the box. Therefore, the beaver needs to take out all balls on the right side of . The first ball that will be taken out is the one on the far right, which is . The next one will be . After taking out these two balls, the beaver can put the remaining balls into the box again in the order given in the question. It will put  in first, then , then , and finally .

Comments

Here is another problem where a visual representation can be really helpful. You can draw the box of balls each time a ball is added or removed.

Connections to Computer Science

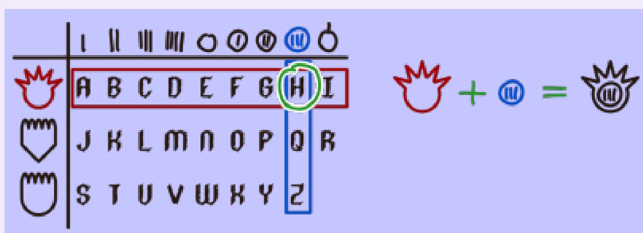
In this problem, you are asked to keep track of which balls are in the box as you consider different things the beaver can do. The key property is that the last ball put in the box is the first ball that must be taken out of the box if any other balls are to be removed. We say that this follows the *last-in first-out* or *LIFO* principle. In computer science, a collection of items that follows this principle is called a *stack*. It is one fundamental way that a collection can be stored.

Ancient Code (BCC Grade 5/6 2019)

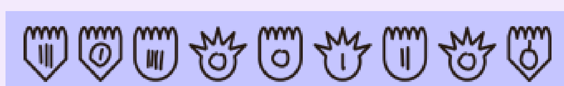
Story

Beaver Cleveria discovered a table of symbols carved in wood.

After studying the table, Cleveria figures out that it is an ancient code. The symbol assigned to a row and the symbol assigned to a column are combined to form a single image. This image is the code for the letter where that row and column meet. For example, the letter H is encoded as shown:



Later, Cleveria sees the following coded message on a tree:



Question

What is the message?

(A) LOVEWATER (B) SLEEPDAYS (C) LOVEMYSUN (D) CAREFORME

Solution - Ancient Code (BCC Grade 5/6 2019)

Answer

(A) LOVEWATER

Explanation of Answer

Cleveria can use the table to decode each image in the message.

The first image in the message is . This corresponds to the third column and second row, and so it is the code for "L". This means that the message cannot be Option B nor D.

Cleveria notices that Options A and C begin with the same four letters, so she skips over to the fifth image. The fifth image is  which is the code for "W". This means that the message cannot be Option C and must be Option A.

To verify that the message is in fact LOVEWATER, Cleveria can continue to decode all the remaining images.

Comments

Here is another problem where we can rule out possibilities rather than proving that it is one answer. For example, in this question, we first ruled out Options B and D based on the first letter. Then, since Options A and C were similar, we skipped to a part that was different and managed to rule out Option C. Since all other options were ruled out, we were left with A. And we didn't even need to decode the entire message!

Connections to Computer Science

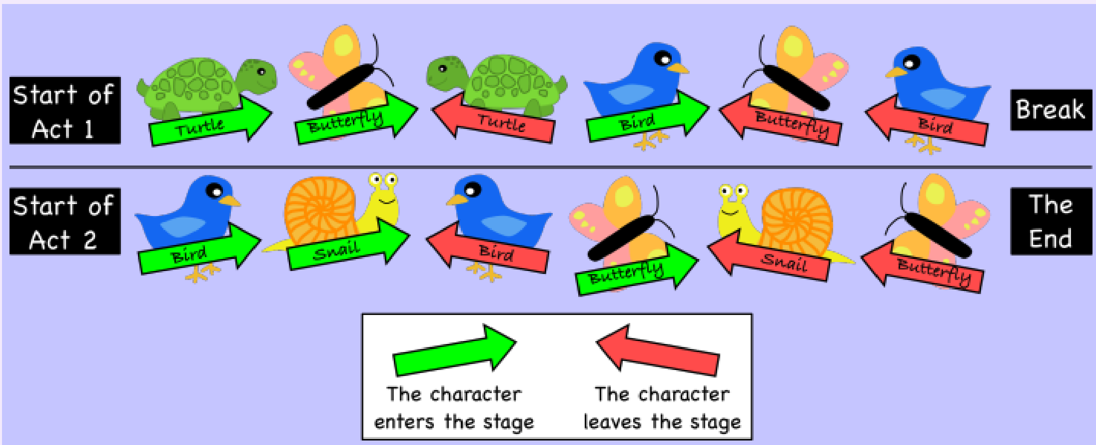
Data security is an important issue in society. One of the methods to protect data from unauthorized persons is to *encrypt* it using a *cipher*. The study of these techniques is called *cryptography* and some historians believe it began approximately 3500 years ago. Some of these ancient systems involved replacing each letter by another letter.

In this problem, new symbols are created for letters of the alphabet in a way that allows us to easily produce coded messages as long as we know the secret table. The secret table is called a *private key*. The security of the system depends on keeping this table secret. Someone who does not have access to the private key might try to break the code by looking at how often each letter appears and trying to observe patterns. Studying possible attacks on a cipher is called *cryptanalysis*.

Theatre Performance (BCC Grade 5/6 2020)

Story

Four characters are in a play. They enter and leave the stage according to the order shown, read from left to right. The play has two acts and one break between the acts.



Question

Which statement is *not* true?

(A) The snail and the butterfly were together on the stage.

(B) The turtle and the bird were together on the stage.

(C) The snail entered the stage after the break.

(D) The snail and the bird were together on the stage.

Solution - Theatre Performance (BCC Grade 5/6 2020)

Answer

(B) The turtle and the bird were together on the stage.

Explanation of Answer

The statement in Option A, "the snail and the butterfly were together on the stage", is true. In Act 2, the snail entered the stage, the bird left the stage, and then the butterfly entered the stage. Therefore, the snail and the butterfly were on the stage at the same time.

The statement in Option B, "the turtle and the bird were together on the stage", is not true. In Act 1, the turtle left the stage right before the bird entered the stage, and the turtle never appeared again in the play.

The statement in Option C, "the snail entered the stage after the break", is true. The snail was the second character to enter the stage in Act 2, right after the bird.

The statement in Option D, "the snail and the bird were together on the stage", is true. At the start of Act 2, the bird entered the stage and then the snail entered the stage. Therefore, the snail and the bird were on the stage at the same time.

Connections to Computer Science

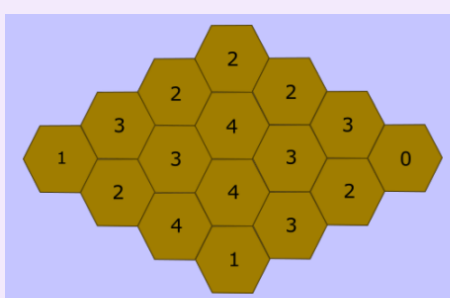
This problem focusses on keeping track of the *state* of a system. The state in this problem is the set of characters which are currently on the stage. Since a character is either on the stage or off the stage, we can keep track of each character using a *binary number*. A binary number is either the number 0 or 1, which we can think of as "off" or "on".

We can then think about the state of the stage as being a list of four binary numbers. For example, if we list the characters in the order (bird, butterfly, snail, turtle), then the state (1, 0, 1, 0) would indicate that bird and the snail are on the stage, and the butterfly and the turtle are off the stage.

Beehive (BCC Grade 5/6 2017)

Story

A bear studies how many hexagons in a honeycomb contain honey. For each hexagon, the bear records how many other hexagons touching this hexagon contain honey. So this number could be 0, 1, 2, 3, 4, 5 or 6. The results of the bear's study are below.



Question

How many hexagons contain honey?

(A) 7 (B) 8 (C) 9 (D) 10

Solution - Beehive (BCC Grade 5/6 2017)

Answer

(C) 9

Explanation of Answer

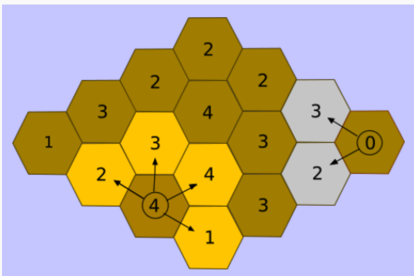
One way to solve this task is to start from the cells for which the content of their adjacent cells is known.

This is the case for two cells, which have a circle in the picture below:

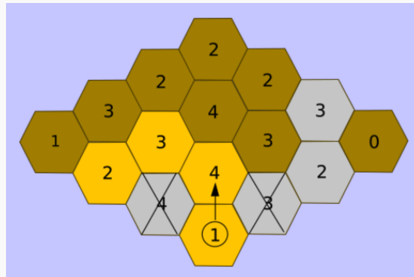
All the adjacent cells of the cells with a 0 are empty.

All the adjacent cells of the circled cell with a 4 are full.

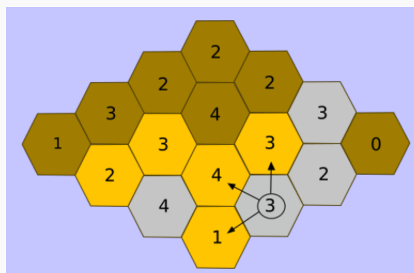
We will use yellow to represent honey in a hex, grey to represent no honey, and brown for undetermined.



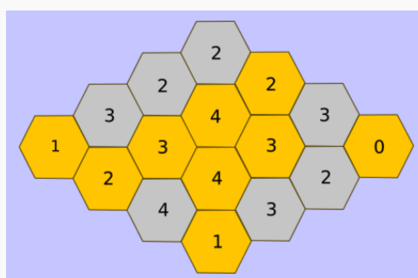
After that, we can easily find some other cells, near the zone that we have changed, for which the content of their adjacent cells is now known. For instance the cell with a circle in the picture below, has only 1 full adjacent cell. It is the one already colored in yellow, just above it. So its other adjacent cells are empty.



Then, the cell with a 3 on it that we just coloured grey, must have exactly 3 full adjacent cells. We have already two of them on its left and the one on the right is empty, so the one above it must be full.



We can go on from one adjacent cell to the others, in order to cover all the beehive. So, there are nine cells in total that contain honey.



Comments

Our interctive online version was really useful at keeping track of which hexagons contained honey. A drawing would work just as well here and be much simpler than trying to keep all the information in your head.

Connections to Computer Science

This task concerns *logic* and *inference*.

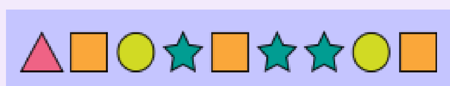
The logic in this problem is to understand that, for example, a cell marked as 0 means that none of its neighbours contain honey. From this fact, we can infer further facts, and build up a solution to the larger problem.

The method of building a solution using inference is what is called a *bottom-up algorithm*: we start with a solution to a very small part of problem (the "bottom" of the problem) and then build up larger and larger solutions until we have solved the entire problem (the "top" of the problem).

Shapes (BCC Grade 5/6 2021)

Story

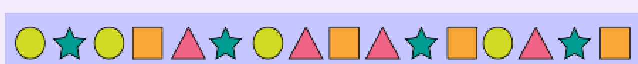
Here is a line of shapes.



The line has a run of stars of length 2. A run is an unbroken chain of identical shapes. Ali likes to create long runs by changing shapes. For example, if Ali changes the middle square to a star in the line above, then he can create a longer run of length 4.

Question

Suppose Ali chooses and changes exactly 3 of the 16 shapes in the following line:



What is the length of the longest possible run that Ali can create?

(A) 4 (B) 5 (C) 6 (D) 7

Solution - Shapes (BCC Grade 5/6 2021)

Answer

(C) 6

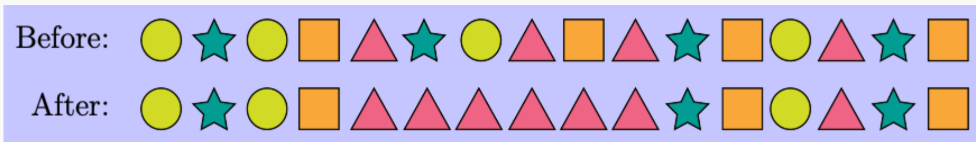
Explanation of Answer

To show that Option C is correct, we need to prove two things:

(1) that a run of length 6 is possible, and

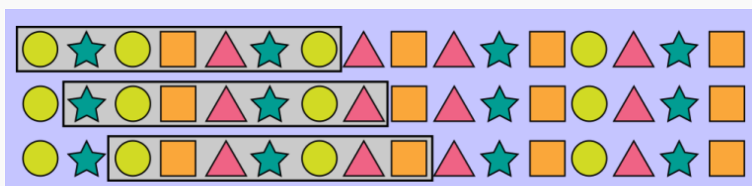
(2) that a run with length greater than 6 is not possible.

A run of length 6 can be made by changing the second star, third circle, and second square to triangles, as shown.



To prove that a run with length greater than 6 is not possible, consider what it would mean if there was a longer run. It would mean there is a run of length 7. Remember that Ali changes only 3 shapes. Therefore, any run of length 7 must come from an unbroken chain of length 7 with 4 identical shapes in it.

There are ten unbroken chains of 7 shapes in the original sequence, the first few of which are shown below.



As we can see, none of these unbroken chains of length 7 have 4 identical shapes in them. Convince yourself that this is also true for the remaining unbroken chains of length 7. This means that it is not possible to have a run of length 7.

We have shown there are 6 shapes in the longest possible run.

Connections to Computer Science

This task is related to finding a *longest substring* that matches some given criteria. A *substring* is a sequence of adjacent characters, such as “ubs” within “substring”.

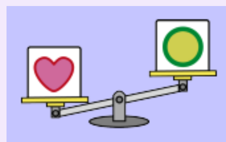
One of the most studied applications of the longest substring in computer science is the *longest common substring* between two strings. Finding the longest common substring is used in plagiarism detection to determine if a document contains a significant amount of copied material.

Weighing Boxes (BCC Grade 5/6 2020)

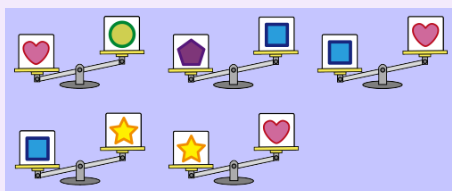
Story

There are five boxes, each featuring a different shape, and each having a different mass. Using a scale we can compare the masses of two boxes.

For example, the following scale shows that  is heavier than :

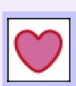
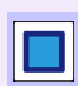


Five comparisons were made, and the results are shown on the following scales:



Question

If we arrange the boxes in order from heaviest to lightest, which box would be in the middle?

(A)  (B)  (C)  (D) 

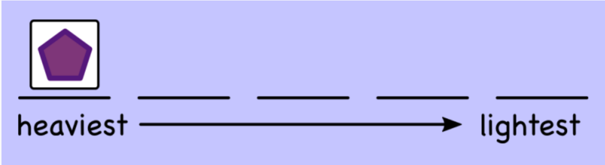
Solution - Weighing Boxes (BCC Grade 5/6 2020)

Answer

(B) 

Explanation of Answer

We can start by arranging the boxes in order from heaviest to lightest. The heaviest box will always appear on the lower part of the scale. Looking at all five boxes and all five comparisons, the pentagon box is the only box that meets this criteria. All the other boxes are on the upper part of the scale in at least one comparison. So the pentagon box is the heaviest box.



Similarly, the lightest box will always appear on the upper part of the scale. Looking at all five boxes and all five comparisons, the circle box is the only box that meets this criteria. All the other boxes are on the lower part of the scale in at least one comparison. So the circle box is the lightest box.

Now we are left with the square, heart, and star boxes. Looking at the bottom two comparisons, we see that the star box is lighter than the square box, but heavier than the heart box. That means the star box must be in between the square box and the heart box.

Therefore, the star box is in the middle when we arrange the boxes in order from heaviest to lightest.

Connections to Computer Science

This problem involves *sorting* elements. The particular *sorting algorithm* that was used in the explanation was *selection sort*. Selection sort finds the heaviest/largest value by comparing the currently known heaviest/largest value to all other values. In this problem, each different scale arrangment is a different *comparison*, and sorting algorithms try to minimize the total number of comparisons.

Problem Set Solutions

1. **Strawberry (BCC Grade 5/6 2021)**

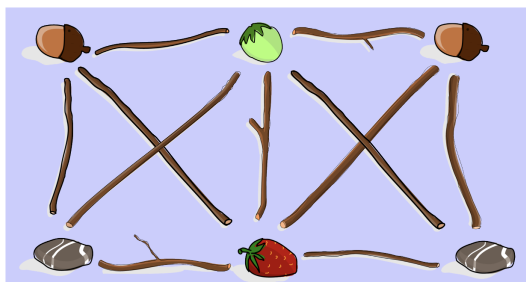
   Anja makes a design on the ground using the following four types of objects.

   She then places sticks in her design according to her very important rule:

   Sticks cannot be placed between objects that are the same type.

   Here is Anja's completed design:

   Suddenly a bird swoops in and eats the ! Anja would like to avoid having this happen again.

   If possible, Anja would like to replace the  with a different type of object, and without moving any sticks. Without breaking her very important rule, which object can Anja replace the  with?

   (A)  (B)  (C)  (D) It is not possible. Only a  could go there.

   Strawberry (BCC Grade 5/6 2021) Solution

   (D) It is not possible. Only a  could go there.

   Unfortunately, Anja is not going to be able to replace the strawberry with a different type of object, without breaking her very important rule.

   In Anja's original design, the strawberry  had sticks between it and every other type of object. Changing the strawberry to anything other than another strawberry would force a stick to exist between two objects of the same type.

2. **Rare Mushrooms (BCC Grade 5/6 2020)**

Colby wants to take a picture of a rare mushroom. To determine whether or not a mushroom is rare, Colby assigns points to the stem and cap according to the following table:

Points for Stem		Points for Cap		
Plain	Layered	Dotted	Horned	Striped
0 points	2 points	1 point	3 points	4 points

A mushroom that scores 5 points or more is rare and a mushroom that scores less than 5 points is not rare.

Which one of the following four mushrooms is rare?

(A)  (B)  (C)  (D) 

Rare Mushrooms (BCC Grade 5/6 2020) Solution

(B) 

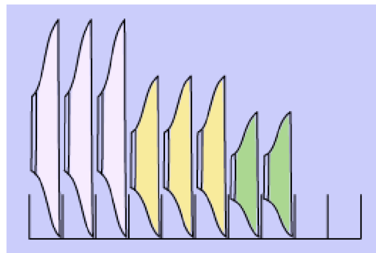
The following table computes the score for each of the four mushrooms:

Mushroom	Stem Score	Cap Score	Total Score	Result
(A) 	Plain (0 points)	Dotted (1 point)	1 point	Not rare
(B) 	Layered (2 points)	Horned (3 points)	5 points	Rare
(C) 	Layered (2 points)	Dotted (1 point)	3 points	Not rare
(D) 	Plain (0 points)	Striped (4 points)	4 points	Not rare

Therefore, the only rare mushroom is the mushroom in Option B.

3. **Plates (BCC Grade 5/6 2019)**

A beaver believes plates are only arranged properly if all the large plates are on the left, followed by all the medium plates, followed by all the small plates. For example, the beaver believes the three large plates, three medium plates, and two small plates shown are arranged properly.



The beaver would like to add a large plate and arrange them properly.

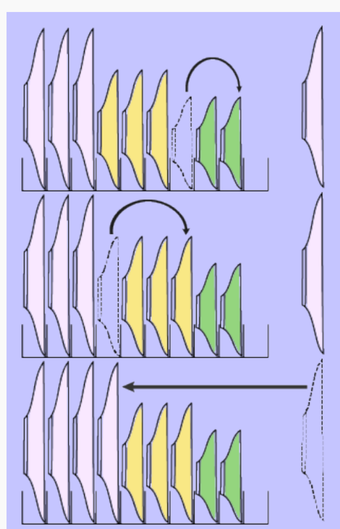
Of the eight original plates, what is the fewest number of plates that must be moved?

(A) 2 (B) 3 (C) 4 (D) 5

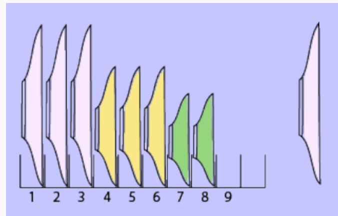
Plates (BCC Grade 5/6 2019) Solution

(A) 2

The task can be solved by moving two of the original plates. The sequence of moves is shown below:



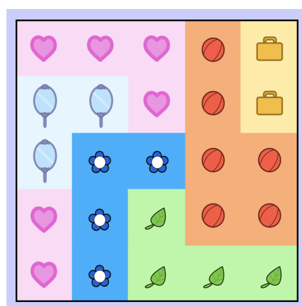
To see that the task cannot be done by moving only one of the original plates, consider the picture below:



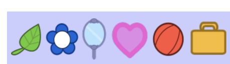
Once the plates are arranged properly, there must be a medium plate in slot 7, and a small plate in slot 9. Neither of these conditions are satisfied initially, which means at least two of the original plates must move.

4. **Wallpaper (BCC Grade 5/6 2017)**

Robyn covers a wall with six overlapping rectangular sheets of wallpaper as shown. Each sheet of wallpaper is designed using a different image in a repeating pattern.



What is the order of the wallpaper pieces from the one placed first to the one placed last?

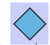
(A)  (B)  (C)  (D) 

Wallpaper (BCC Grade 5/6 2017) Solution

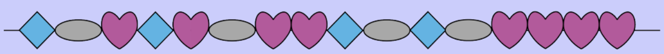
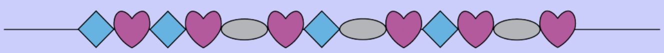
(A) 

The yellow wallpaper with the briefcases is the only wallpaper that isn't cut off by another one, so that must be the last one. You can see the suitcase cuts off the basketball wallpaper, that the basketball wallpaper cuts off the leaf wallpaper, the leaf wallpaper cuts off the flower wallpaper, the flower wallpaper cuts off the mirrors and the mirrors cut off the hearts.

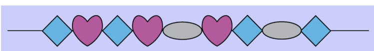
5. **Necklaces (BCC Grade 5/6 2021)**

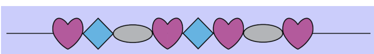
A jeweller makes necklaces with hidden messages by replacing each letter of the alphabet with a bead pattern. Bead patterns are made using heart  and diamond  beads, and the same bead pattern always represents the same letter. Letters in a message are separated by oval  beads and messages are read from left to right.

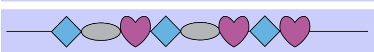
Here are two of the necklaces the jeweller has made along with their hidden messages.

Necklace	Hidden Message
	TRUTH
	CARE

Which of the following necklaces has the hidden message ART?

(A) 

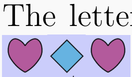
(B) 

(C) 

(D) 

Necklaces (BCC Grade 5/6 2021) Solution

(D) 

The letter A is the second letter in the message CARE, so the bead pattern for A is . The letter R is the second letter in the message TRUTH, so the bead pattern for R is .

The letter T is the first letter in the message TRUTH, so the bead pattern for T is .

Stringing these together from left to right and separating them with oval beads gives the necklace in Option D.

Though not required, we can use this method to find the hidden messages in the other necklaces as well. Option A contains the message CAT, Option B contains the message ARE, and Option C contains the message TAR.

6. Push-Away Parking (BCC Grade 5/6 2019)

In the parking lot shown, each car is either parked in a parking space or in front of two parking spaces.

Cars that are parked in front of two parking spaces may be moved forward or backward in order to allow blocked cars to exit. For example, Car A is not blocked and can exit without any other cars moving; however, Car L is blocked by Car Q. If Car Q is moved, then Car L can exit.

Which car cannot exit its parking space unless two different cars move?

(A) Car G (B) Car H (C) Car I (D) Car B

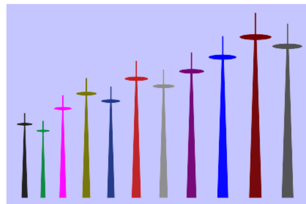
Push-Away Parking (BCC Grade 5/6 2019) Solution

(C) Car I

If Car O moves, then Car G can exit. If Car P moves, then Car H can exit. If Car N moves, then Car B can exit. Car P must move in order for Car I to exit, but Car P cannot move far enough to allow Car I to exit without Car O or Car Q also moving. Therefore, at least two different cars must move in order for Car I to exit its parking space.

7. **Special Towers (BCC Grade 5/6 2019)**

Consider the towers shown.



A tower is special if all towers to the left of it are shorter, and all towers to the right of it are taller.

How many special towers are there?

(A) 1 (B) 2 (C) 3 (D) 4

Special Towers (BCC Grade 5/6 2019) Solution

(C) 3

Figures 1 and 2 show that the eighth tower is special. A tower is special if all the tips of the towers to the left are within the coloured rectangle (see Figure 1); and if all the tips of the towers to the right are outside the coloured rectangle (see Figure 2).

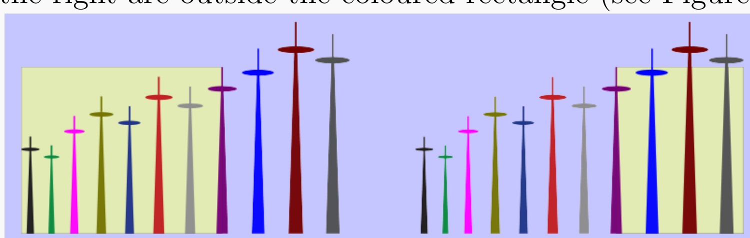


Figure 1 Figure 2

One possible way to find all the special towers is to move through all the towers, one by one, and mark a tower if all to the left of it are shorter (see Figure 3). Then make a similar second pass, marking a tower if all to the right of it are taller (see Figure 4).

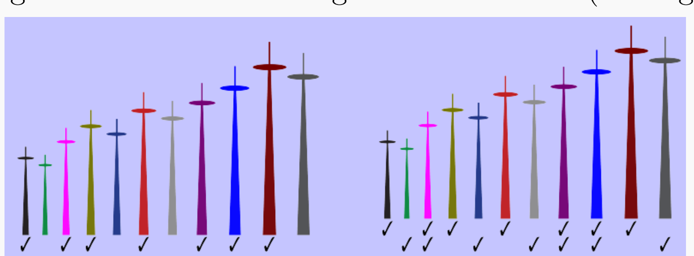
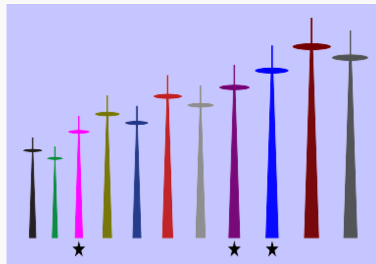


Figure 3 Figure 4

All towers marked twice are special towers.



8. **Paintings (BCC Grade 5/6 2021)**

Paintings are brought to a warehouse for inspection before they are delivered to museums. The paintings are stacked on top of each other. When a painting arrives at the warehouse, it is put on top of the stack. When a delivery person departs with a painting, they take the painting from the top of the stack.

Records are kept of all paintings arriving at the warehouse and departing from the warehouse:

Arrivals		*Departures*	
Time	**Painting**	**Time**	**Delivery Person**
11:40	Beavers on the Grass	12:25	Pia
12:15	Happy Beaver	13:35	Raz
12:55	Sun and Moon	14:35	Stu
13:30	Enchanted Forest	14:40	Quy
14:18	Oak and Birch	15:20	Raz

Which delivery person took "Sun and Moon" to a museum?

(A) Pia (B) Quy (C) Raz (D) Stu

Paintings (BCC Grade 5/6 2021) Solution

(B) Quy

There are two important types of events: a painting is put on the stack and a delivery person takes a painting from the top of the stack. From the two given tables, we build a single new table that displays both types of events together and the state of the stack after each event. We add rows to the table in chronological order stopping when we learn which delivery person took "Sun and Moon" to a museum.

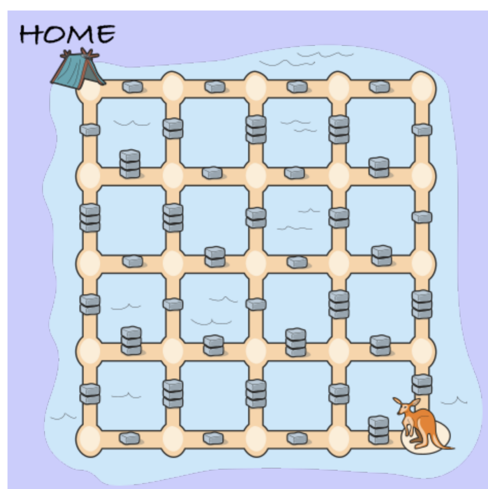
Time	Event	Paintings on the stack
11:40	Arrival of Beavers on the Grass	Beavers on the Grass
12:15	Arrival of Happy Beaver	Happy Beaver Beavers on the Grass
12:25	**Pia** takes Happy Beaver	Beavers on the Grass
12:55	Arrival of Sun and Moon	Sun and Moon Beavers on the Grass
13:30	Arrival of Enchanted Forest	Enchanted Forest Sun and Moon Beavers on the Grass
13:35	**Raz** takes Enchanted Forest	Sun and Moon Beavers on the Grass
14:18	Arrival of Oak and Birch	Oak and Birch Sun and Moon Beavers on the Grass
14:35	**Stu** takes Oak and Birch	Sun and Moon Beavers on the Grass
14:40	**Quy** takes Sun and Moon	Beavers on the Grass

9. **Jumping Kangaroo (BCC Grade 5/6 2020)**

Kanga Roo is jumping home along the vertical and horizontal paths. Kanga jumps over exactly one pile of bricks with each jump. Kanga cannot jump over brick piles that have a height of 3 bricks.

If Kanga wants to jump home using the fewest jumps possible, how many jumps must Kanga make?

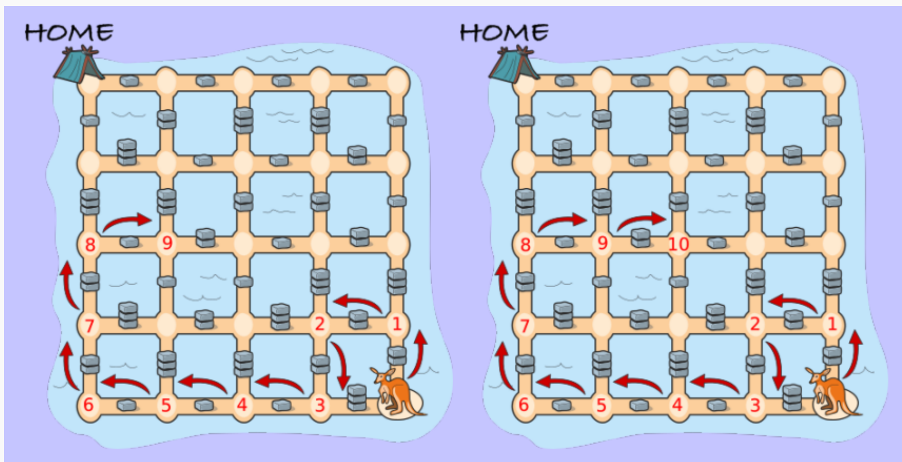
(A) 8 (B) 13 (C) 14 (D) 16



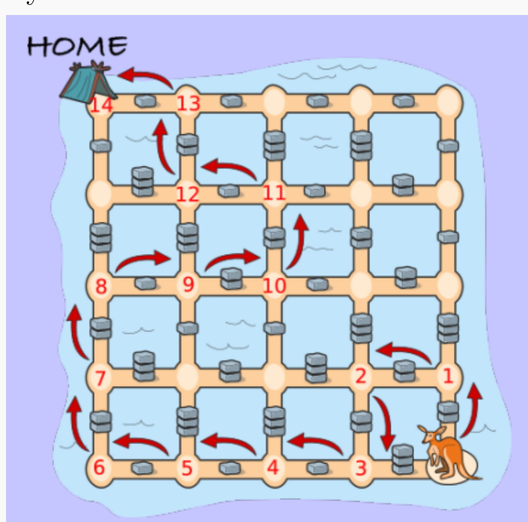
Jumping Kangaroo (BCC Grade 5/6 2020) Solution

(C) 14

Kanga does not have a choice for the first 9 jumps because of the positions of the brick piles that are 3 bricks high. After the 9th jump, Kanga can either jump right OR down-right-up. Both bring Kanga to the same place on the paths. Since one jump is fewer than three, Kanga jumps right.



Now notice that any jump either up or left brings Kanga closer to home, and any jump either right or down brings Kanga further from home. As long as Kanga jumps only up and left, the jumps will be minimal. As it turns out, from Kanga's current position this is possible to do all the way home.



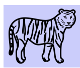
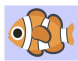
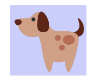
Kanga can get home in as few as 14 jumps.

10. **Toy Storage (BCC Grade 5/6 2017)**

Tom has two types of toys: animal toys and vehicle toys. Tom fills three boxes by putting three toys in each box. As long as there is room, he puts vehicles into box A, animals with striped bodies into box B, and animals with spotted bodies into box C. However,

- Anytime he tries to put a toy in box A and it is full, he then tries to put the toy in box B.
- Anytime he tries to put a toy in box B and it is full, he then tries to put the toy in box C.
- Anytime he tries to put a toy in box C and it is full, he then tries to put the toy in box A.

Tom puts the following nine toys into boxes in the following order:

(1)  (2)  (3)  (4)  (5)  (6)  (7)  (8)  (9) 

Where does Tom put the dog and zebra?

(A) Tom puts the dog in box C, and the zebra in box B.

(B) Tom puts both in box A.

(C) Tom puts both in box B.

(D) Tom puts both in box C.

Toy Storage (BCC Grade 5/6 2017) Solution

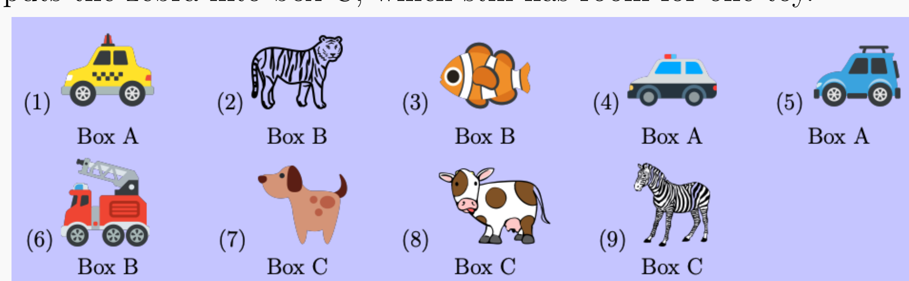
(D) Tom puts both in box C.

For toys 1 to 5, we can simply follow the rules and put them into corresponding box.

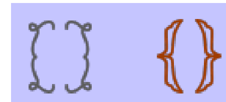
When Tom tries to put the firetruck into box A according to rule (1), he realizes that box A is full. He puts the truck into box B, which still has room for one toy.

The dog and cow have spots so Tom puts them in box C, which still has room for three toys.

When Tom tries to put the zebra into box B according to rule (2), he realizes that box B is full. He puts the zebra into box C, which still has room for one toy.



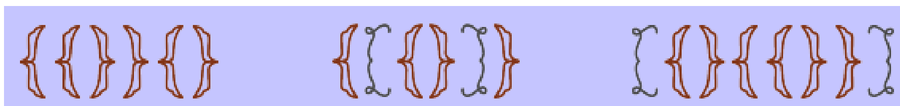
11. Bracket Bracelet (BCC Grade 5/6 2017)



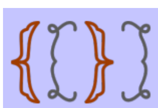
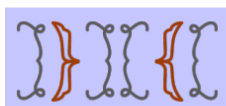
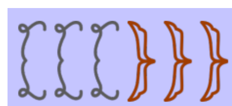
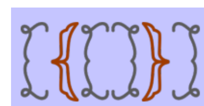
A jewelry shop produces chains used to make bracelets. The chains are built by continually adding matching pairs of bracket-shaped ornaments. There are two types of pairs, as shown on the right.

After choosing a starting pair, a second pair is either added to the end of the chain or inserted between the previously added pair. This process can be repeated any number of times.

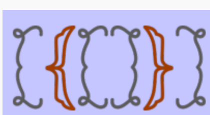
Examples of three different chains that can be produced are shown below.



Which of the following chains can also be produced?

(A)  (B)  (C)  (D) 

Bracket Bracelet (BCC Grade 5/6 2017) Solution

(D) 

They started off with two bracelets, placed a pair in between and then placed a third pair between the second pair.

All other bracelets cannot be constructed according to the methods stated in the task:

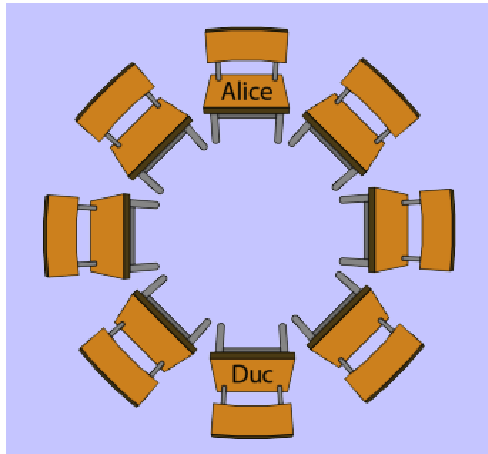
(A) A bronze block is opened by the left bronze ornament, then a silver block is opened by the left silver ornament, then the bronze block is closed by the right bronze ornament. However, this cuts off the silver block as it hasn't been closed yet;

(B) The first ornament is a right silver ornament, but we have not yet opened a silver block so we can't yet close one;

(C) There are 3 left silver ornaments and then 3 right bronze ornaments, so there is no pairing between the ornaments.

12. **Seating Plan (BCC Grade 5/6 2019)**

Berto and seven of his friends are sitting in a circle. They are all facing inwards.



We know the following facts about where the friends are sitting:

1. Alice is sitting directly across from Duc, as shown.
2. Greta and Eugene are both sitting beside Haakim.
3. Franny is not sitting beside Alice or Duc.
4. There is someone who is sitting next to both Greta and Chika.
5. Eugene is beside Duc, on Duc's left.

Which of these orders of friends, in a clockwise manner, is correct?

(A) Alice, Berto, Greta, Duc, Chika, Eugene, Franny, Haakim
(B) Alice, Greta, Haakim, Eugene, Duc, Berto, Franny, Chika
(C) Alice, Chika, Franny, Berto, Duc, Eugene, Haakim, Greta
(D) Alice, Haakim, Eugene, Greta, Duc, Franny, Berto, Chika

Seating Plan (BCC Grade 5/6 2019) Solution

(C) Alice, Chika, Franny, Berto, Duc, Eugene, Haakim, Greta

Alice and Duc are sitting across from each other, so there are three people between them in either direction. Part of the seating arrangement is

Alice, _____, _____, _____, Duc, _____, _____, _____.

We know that Eugene is sitting next to Duc on Duc's left, so we can place Eugene on Duc's left. Remembering that the friends are facing inwards, Eugene should go in our clockwise seating like this:

Alice, _____, _____, _____, Duc, Eugene, _____, _____.

Haakim is sitting between Greta and Eugene, so Haakim and Greta must be sitting as follows:

Alice, _____, _____, _____, Duc, Eugene, Haakim, Greta.

Franny must be in the middle of the three remaining positions in order to satisfy the third condition (Franny is not beside Alice or Duc), which gives us the following:

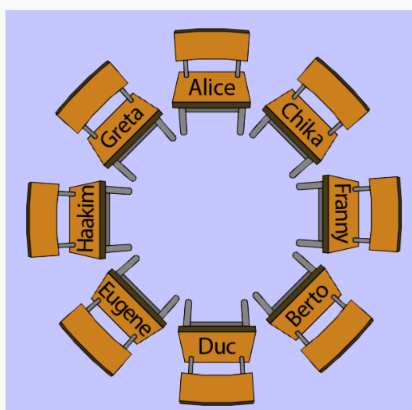
Alice, _____, Franny, _____, Duc, Eugene, Haakim, Greta.

There is one person between Greta and Chika, which means Chika must go between Alice and Franny:

Alice, Chika, Franny, _____, Duc, Eugene, Haakim, Greta.

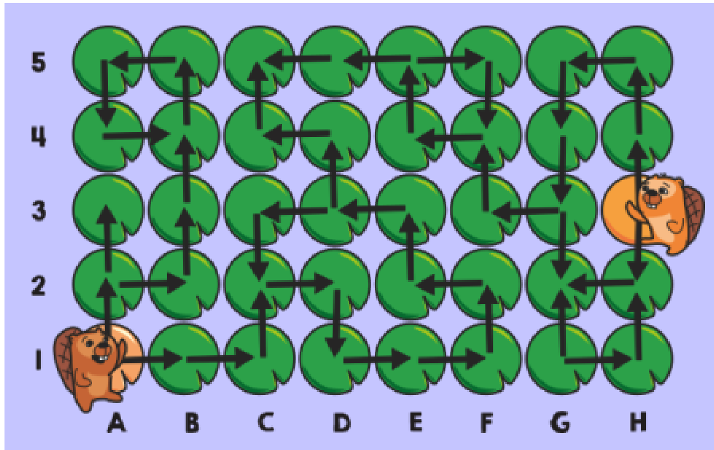
Berto goes in the final position:

Alice, Chika, Franny, Berto, Duc, Eugene, Haakim, Greta.



13. **Do They Meet? (BCC Grade 5/6 2021)**

On Lake Castor, lilypads are arranged in a grid, where rows are numbered from 1 to 5, and columns are labelled from A to H. Beaver Bob starts on pad A1 (in the bottom-left corner), and Beaver Nora starts on pad H3.



The beavers can move from one lilypad to another lilypad only if they are following an arrow. The beavers do not necessarily move at the same speed.

Which of the following statements is true?

(A) The beavers will never meet.

(B) The beavers could meet on pad C2.

(C) The beavers could meet on pad F4.

(D) The beavers could meet on pad C5.

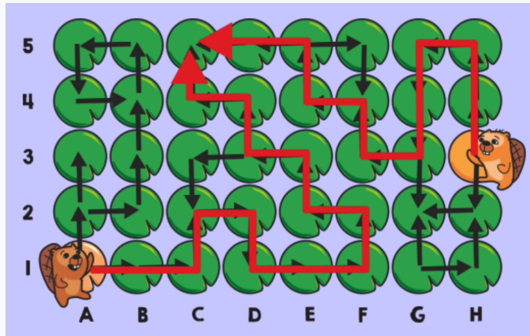
Do They Meet? (BCC Grade 5/6 2021) Solution

(D) The beavers could meet on pad C5.

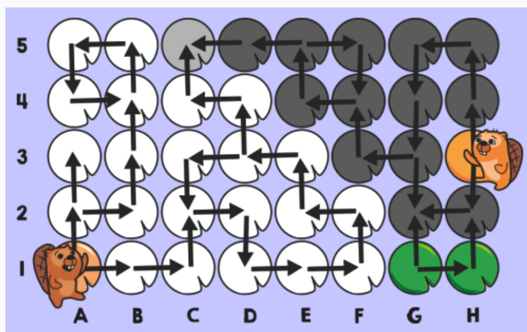
At his starting position, Bob has two options. If he goes "up", then he will either run into the dead end at A3 or get stuck in the loop that begins at B4. If he goes "right", he will follow the arrows to D3. At D3 he can either go to C3 which goes into a loop that will take him back to D3, or go to D4, which leads to a dead end at C5.

Nora also has two options at the start. If she goes "down", she will run into the dead end at G2. If she goes "up", she will reach G3. From there she can either run into the G2 dead end again, or go "left" and reach E5. There she can either go into a loop that will take her back to E5, or reach another dead end at C5.

Since Bob can reach C5 as well, the beavers could meet at C5 using the routes shown below.



But this does not yet fully guarantee that they cannot meet at F4 or C2 either. The next picture shows the set of pads that Bob (white) and Nora (dark gray) can reach by following the arrows in any possible way. We can see that C5 is the only pad common to both sets.



14. For more practice, try writing the BCC Grade 5/6 2018 contest as though you were writing the actual contests. That is, no aid, no notes, and with a time limit of 45 minutes. Here is the link for the contest: https://cemc.uwaterloo.ca/contests/past_contests/2018/2018BCCContest5_6.pdf.

BCC Grade 5/6 2018 Solution

Link for contest solutions: https://cemc.uwaterloo.ca/contests/past_contests/2018/2018BCCContestSolutions5_6.pdf